A Walk Through the Virtual Mathematics Exploratorium

1. Introduction

The *Virtual Mathematics Exploratorium* (VME) is the name of a mathematical knowledge manifold that is being constructed by Ambjörn Naeve using Conzilla. The concepts are being described with metadata and filled with content components according to the general design principles for knowledge manifolds that have been outlined above. Moreover, filters are created that allow the selective viewing of the content – based on different aspects and levels of difficulty[1]. The idea is that teachers should be able to browse through the exploratorium and find components that cover the relevant aspects of the topics they are interested in at the appropriate level of complexity. We are presently working on a component composition environment, where components will be easily integrated into customized learning modules in a work process that will support both a single "component composer" as well as a team of such composers that are involved in different forms of collaborative curriculum design.[2]

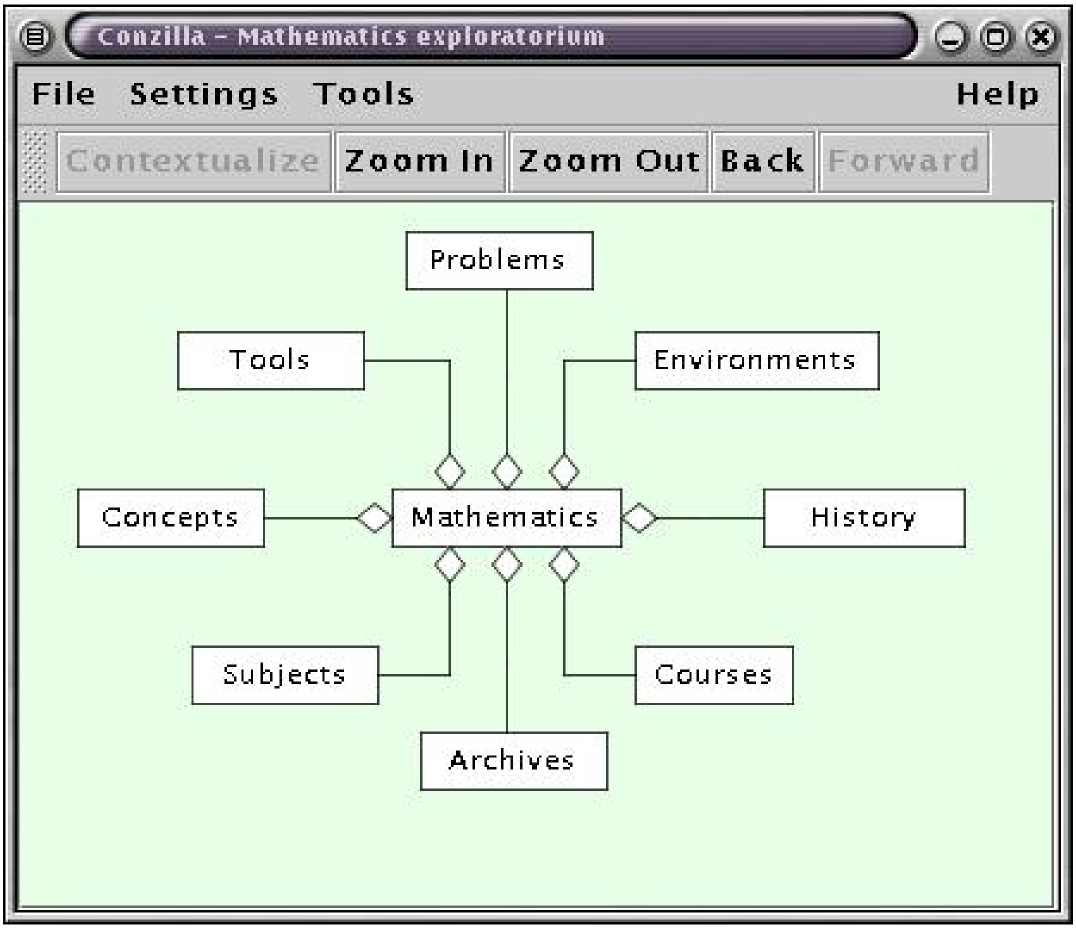

Fig. 1: The entry to the Virtual Mathematics Exploratorium.

[1] The ease with which such filters are constructed and modified is a major strength of the Conzilla tool.
[2] This work is the major theme of a doctoral thesis project on knowledge components by Fredrik Paulsson under the supervision of Ambjörn Naeve. This project is described below.

While functioning for teachers and learning module designers as a distributed archive (open repository) of resource components, the VME will also serve as an environment where learners can navigate through the mathematical landscape and explore the topics of their own interest at the level of complexity of their own choice. This is an strategically important functionality of an interactive learning environment that aspires to support inquiry based learning. See [1] and [2] for more detailed discussions of this topic.

Figures 1 through 11 present a series of screenshots from a navigational tour of the VME. The entire VME will be available on the web within the near future.

On the top (= entrance) level of the VME, eight different parts of the concept of Mathematics are presented: Concepts, Subjects, Tools, Problems, Environments, History, Courses and Archives (Fig. 1). Of course, parts can be added (or taken out) as the design of the VME evolves.

2. Surfing the context

Double-clicking the Concepts box changes the context and takes us into the context of “Mathematics Concepts” shown in Fig. 2. In the lower right we also see a (slightly shaded) rectangle that displays metadata about the map.

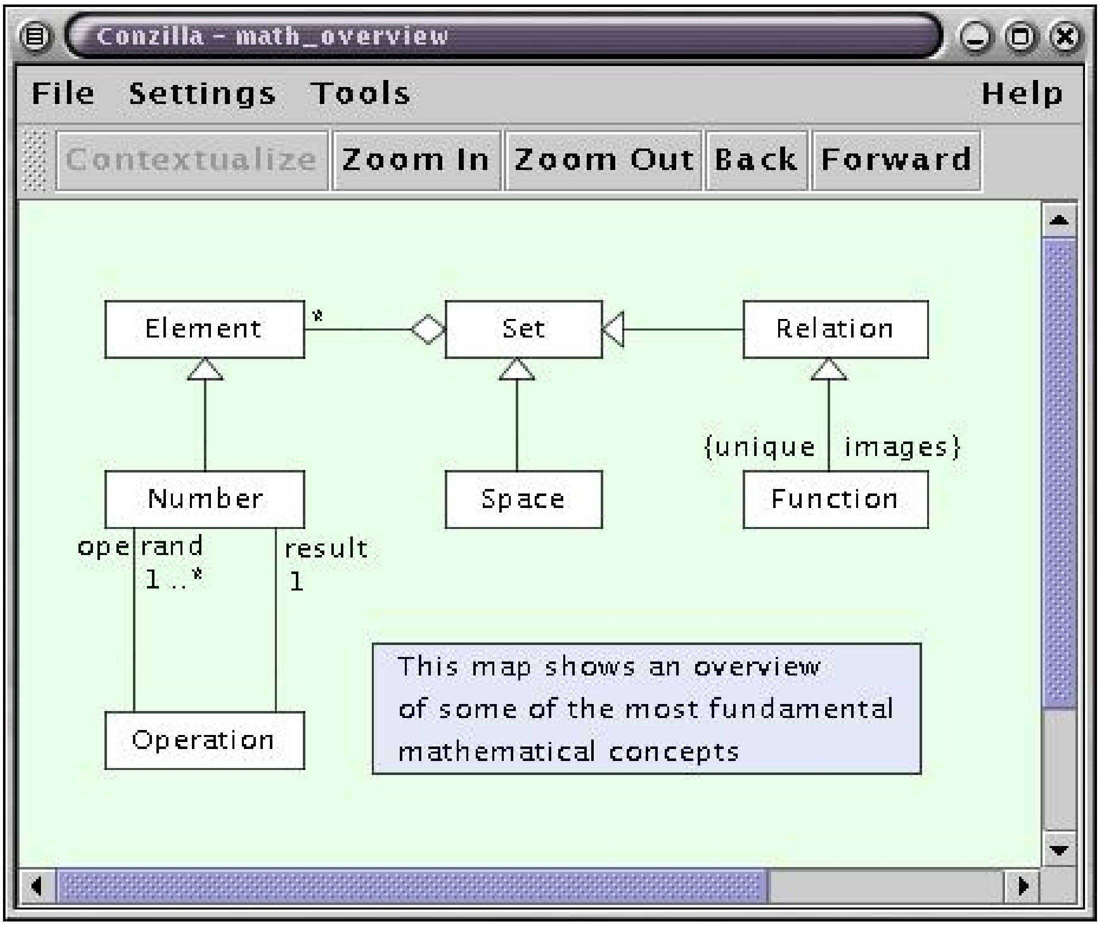

Fig. 2: An overview of mathematical concepts with a metadata description.

Double-clicking the Number-box takes us into the context of diffferent kinds of numbers, which is shown in Fig.3. Here we some of the different kinds (= specializations) of the concept of Number: Natural -, Integer -, Rational -, Real – and Complex number. The map also shows the relationships between these different kinds of numbers: Real numbers are displayed as specializations of (kinds of) Complex numbers, Rational Numbers as specializations of Real numbers, etc. By pointing to the specialization arrows and hitting space bar) we can bring up an explanation (= meta-data) of the structure of the corresponding specialization. As an exmple, pointing to the arrow between Integer and Rational would bring up the explanation: An integer is a rational number with the denominator equal to 1.

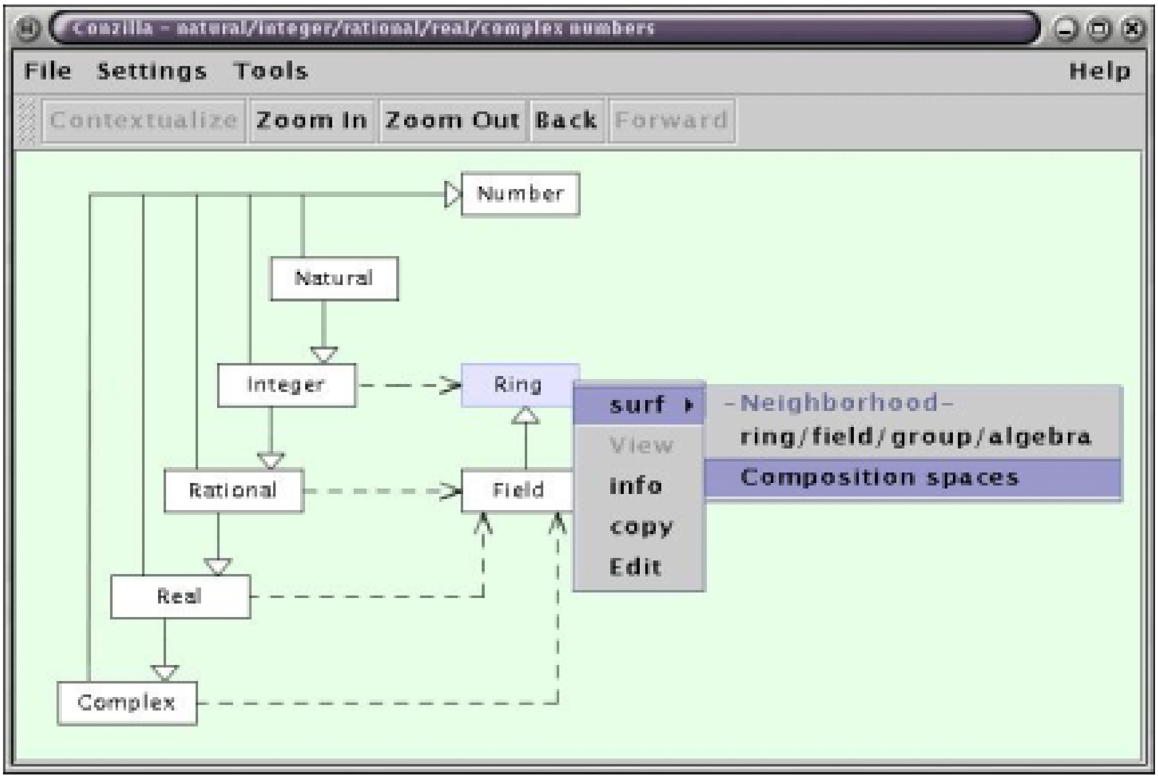

Fig. 3: Surfing the context: Switching to the map **Composition spaces**.

The dashed arrows of Fig. 3 show that the Integers are an example (= instance) of a Ring, while the Rational -, Real - and Complex numbers are examples of Fields. From the figure we also see that Field is a specialization of Ring.

Surfing by double-clicking requires a special map (called a *detailed map*) to be associated with the corresponding concept. If we want to find out more about the Ring concept, we could surf the context in a different (and more general) way. Right-clicking on "Ring" and choosing "surf " displays a list of the *contextual neighborhood* of the Ring concept, i.e. the set of all contexts (= maps) where the concept of Ring appears. Choosing "Composition spaces" from this list brings up the map shown in Fig. 4.

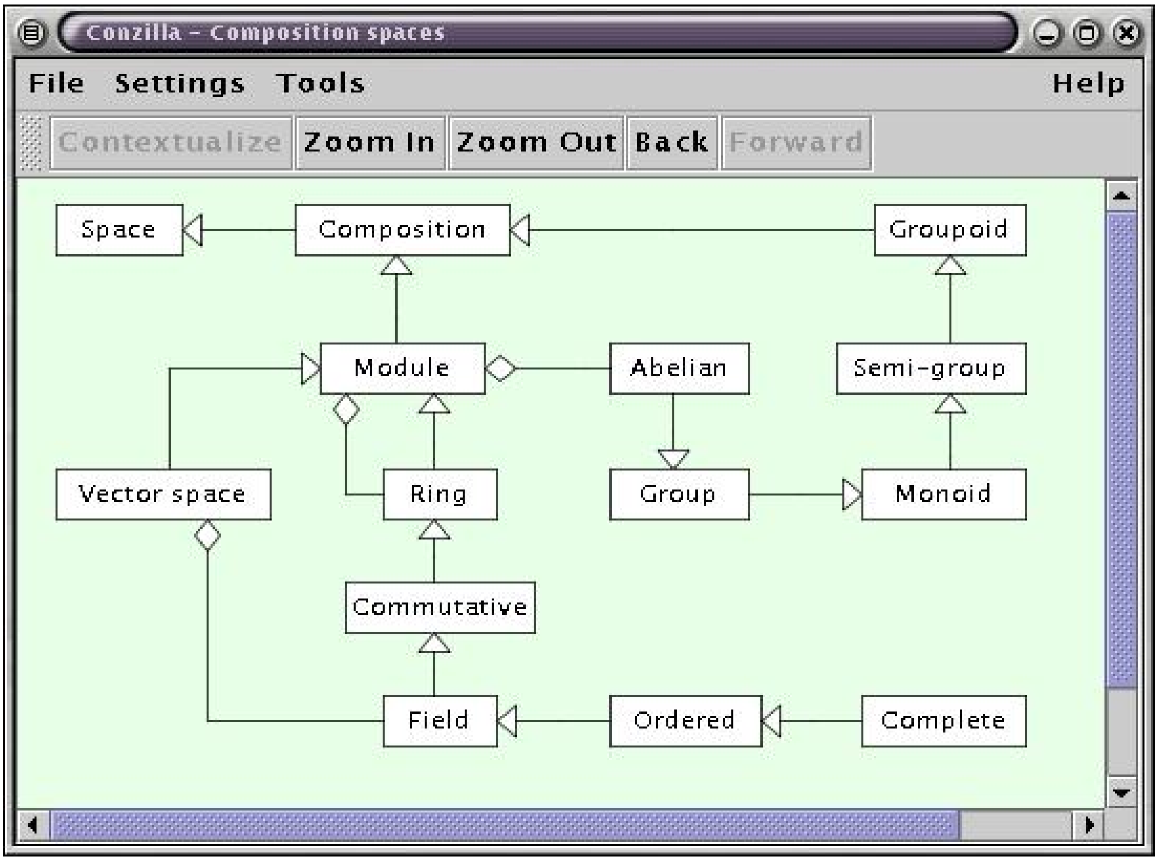

Fig. 4: A concept map showing different kinds of composition spaces.

Here we can see that Groupoids and Modules are special kinds of Composition Spaces, that Vector Spaces and Rings are special kinds of Modules, and that Semi-Group, Monoid, Group, and Abelian Group form a sequence of successive specializations of Groupoid. We also see from the Fig. 4 that a Module contains a Ring and a Vector space contains a Field, which in itself is a special kind of Commutative Ring. Fig. 4 also presents the concepts of Ordered Field and Complete Ordered Field

3. Displaying meta-data descriptions

Pointing to a concept box and hitting the arrow tangents, we can bring up various parts of the metadata list of descriptions of the corresponding concept. By pushing the → key, we can move foreward in the list, and by pushing the ← key we can move backward. Moreover, from any position in the list, by pushing the down-arrow, we can expand the list in the forward direction, and by pusing the up-arrow we can contract it again. And when we have found a piece of the description that we want to emphasize, we can leave it up on the screen and then move the cursor to another concept-box (or conceptual relationship arrow) and start "fishing for meta-data" in the new position. This gives us a lot of flexibility in controlling the simultaneous display of the various pieces of the different description lists for the entire collection of concepts and conceptual relationships that are displayed in the map.

A result of such a “meta-data fishing trip” is shown in Fig. 5. Here we see displayed some of the axioms for the concept of “Vector space” (nr 5-8), one of the axioms for the concept of “Ring” (nr 1), and the definitions of Groupoid, Semi-group, Monoid, Commutative Ring and Field. We emphasize again that the meta-data of the different concepts can be manipulated independently of one another.

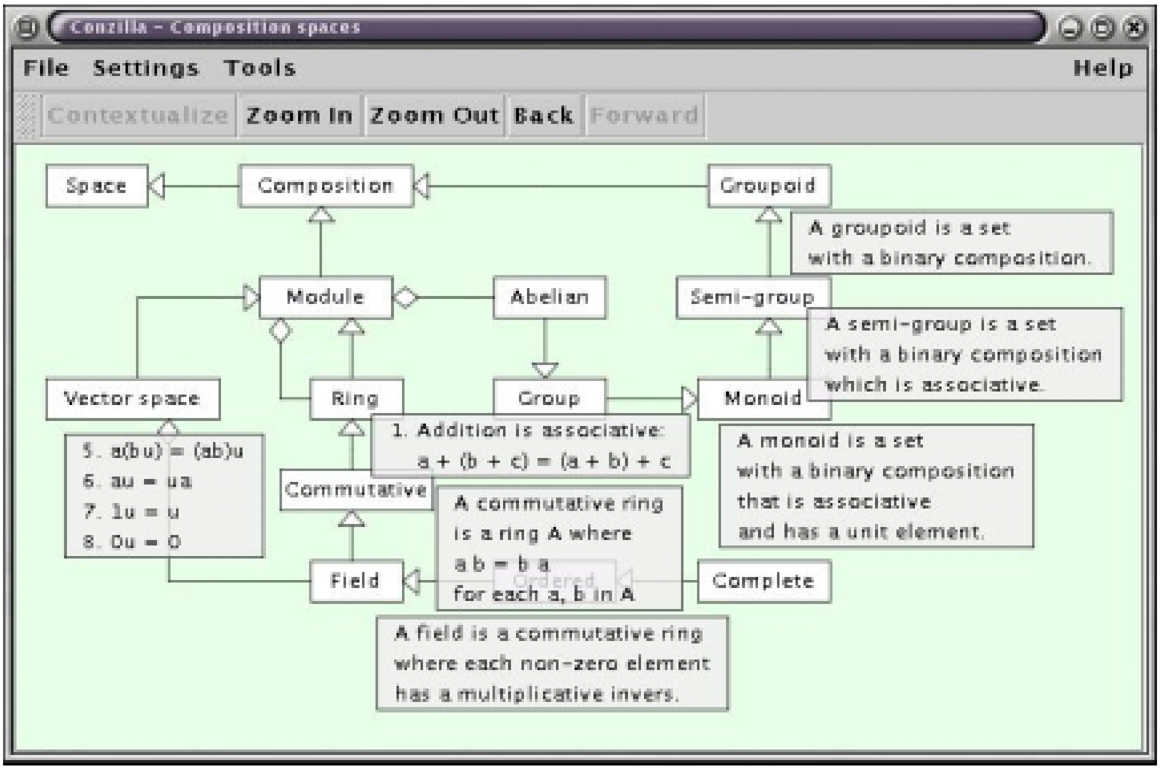

Fig. 5: Using the arrow-keys, we can bring up various pieces of metadata about the different concepts and conceptual relationships.

4. Viewing the list of content components through different filters

While changing the context is effected by choosing “surf” from the pop-up menu, viewing the content is achieved by choosing “view” from the same menu. This brings up the sub-menus that are displayed in Fig. 6. The three choices labeled “Aspects”, “Levels and “Resources” each correspond to different *filters* that display only the content components that are marked with the approporate matching keywords . The choice of “Any”, which has been made in Fig. 6, causes the display of the entire list of components that match *any* of the three filters, while the choice of “Other” would display a list of the components that don’t match any of these filters.

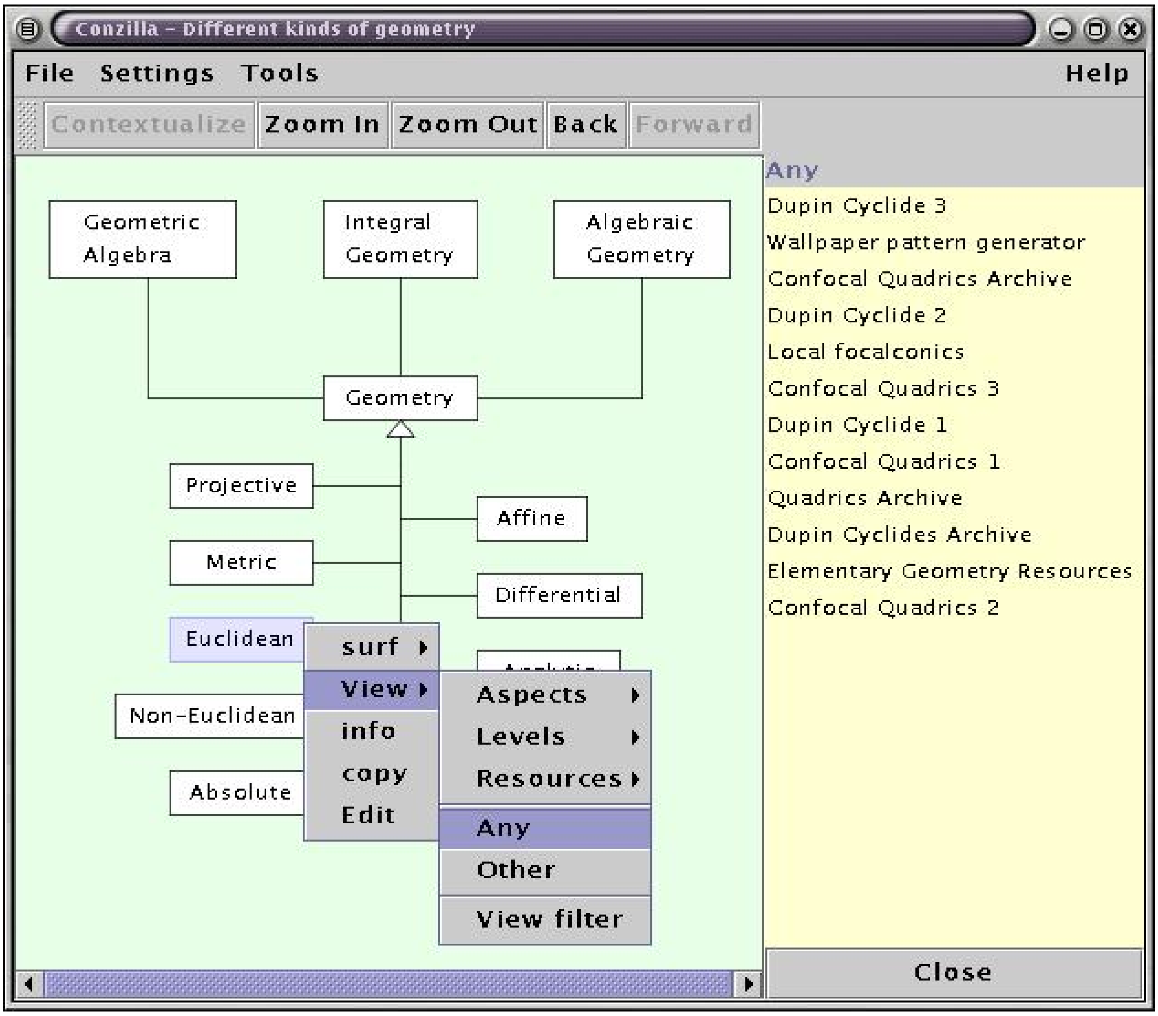

Fig. 6: Viewing the list of content components of the concept "Euclidean Geometry" that match any of the three filters "Aspects", Levels" or "Resources".

The display of a filtered list of content components is illustrated in Fig. 7. Here the content of "Projective Geometry" has been filtered through the Aspects-filter shown in Fig. 8. This filter map has been displayed by choosing "View Filter" from the third menu (= second sub-menu) shown in Fig. 7.

The Aspects-filter is constructed in such a way that each of the chosen aspects (What, How, Where, When, Who, Contact) is connected to each of the chosen levels (Elementary, Intermediate, Advanced, Expert). Hence these levels will appear as sub-filters (and sub-menus) under each one of the aspects. The grayed out parts of the filter menus of Fig. 7 correspond to combinations of keywords that are non-existent, i.e. not presently found among any of the components.

The Levels-filter is the "transpose" of the Aspects-filter, with each of the different aspects now appearing as sub-filters under each of the different levels. Taken together these two filters emulate the matrix-filtering technique discussed in [2].

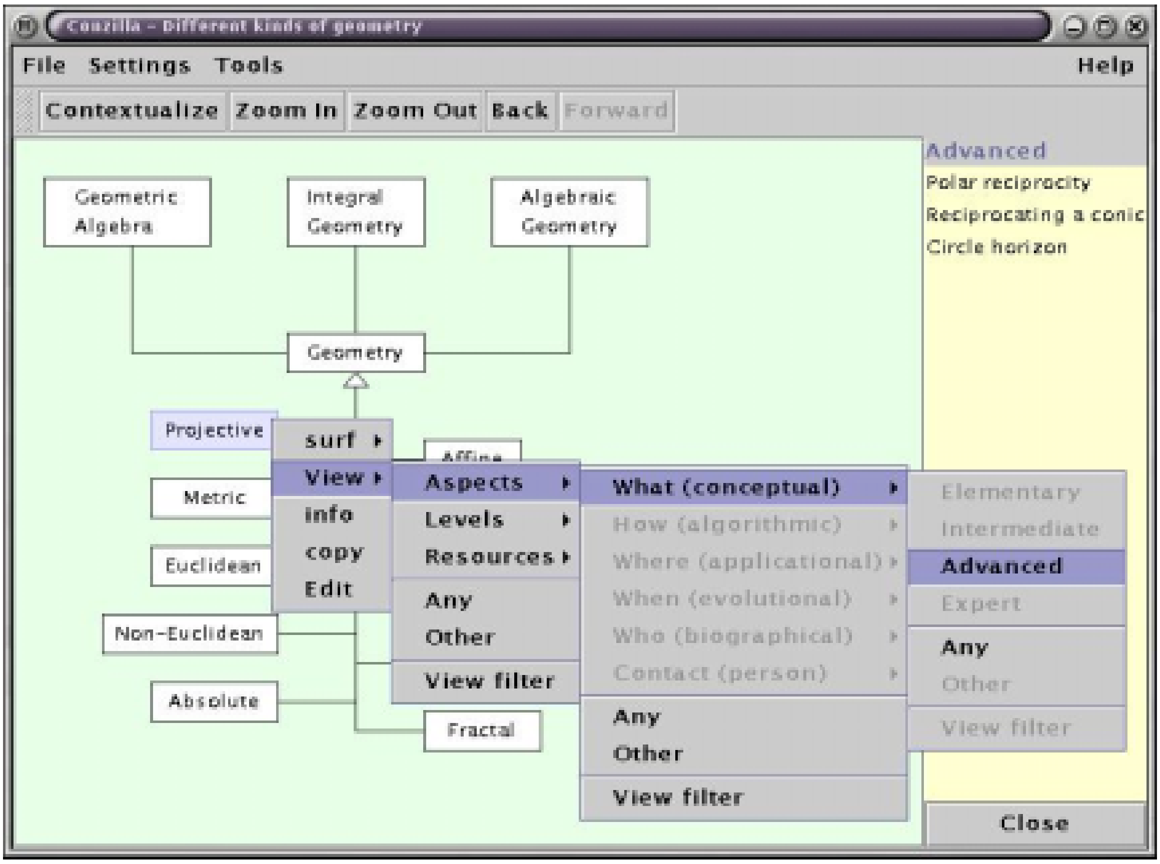

Fig. 7: Looking at the content of “Projective Geometry”, filtered through the aspect “What”, and the level “Advanced”.

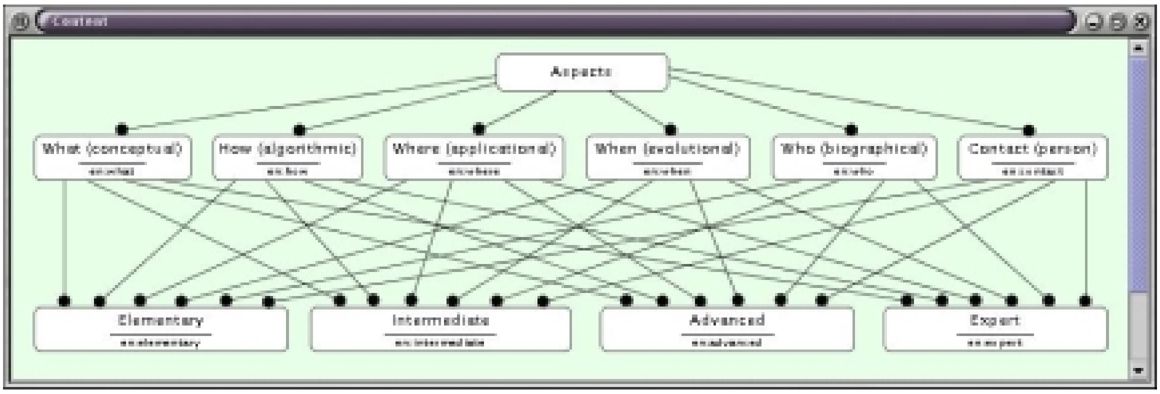

Fig. 8: A filter is controlled by a kind of Conzilla map (called a filter map).

5. Viewing the actual content of a component

Right-clicking on one of the listed content components and choosing “View” from the pop-up menu will display the actual content of this component in the web-browser of the user’s own choice. This is shown in Fig. 9. It is important to point out that the actual location of a component is well encapsulated from the rest of the Conzilla program. Hence a content component could be stored locally (on the user’s own computer) or anywhere on a computer connected to the Internet. Hence the Internet functions as a distributed component archive, with content that can be described (= tagged with meta-data) and associated with (= referenced by) any concept or conceptual relationship created by Conzilla.

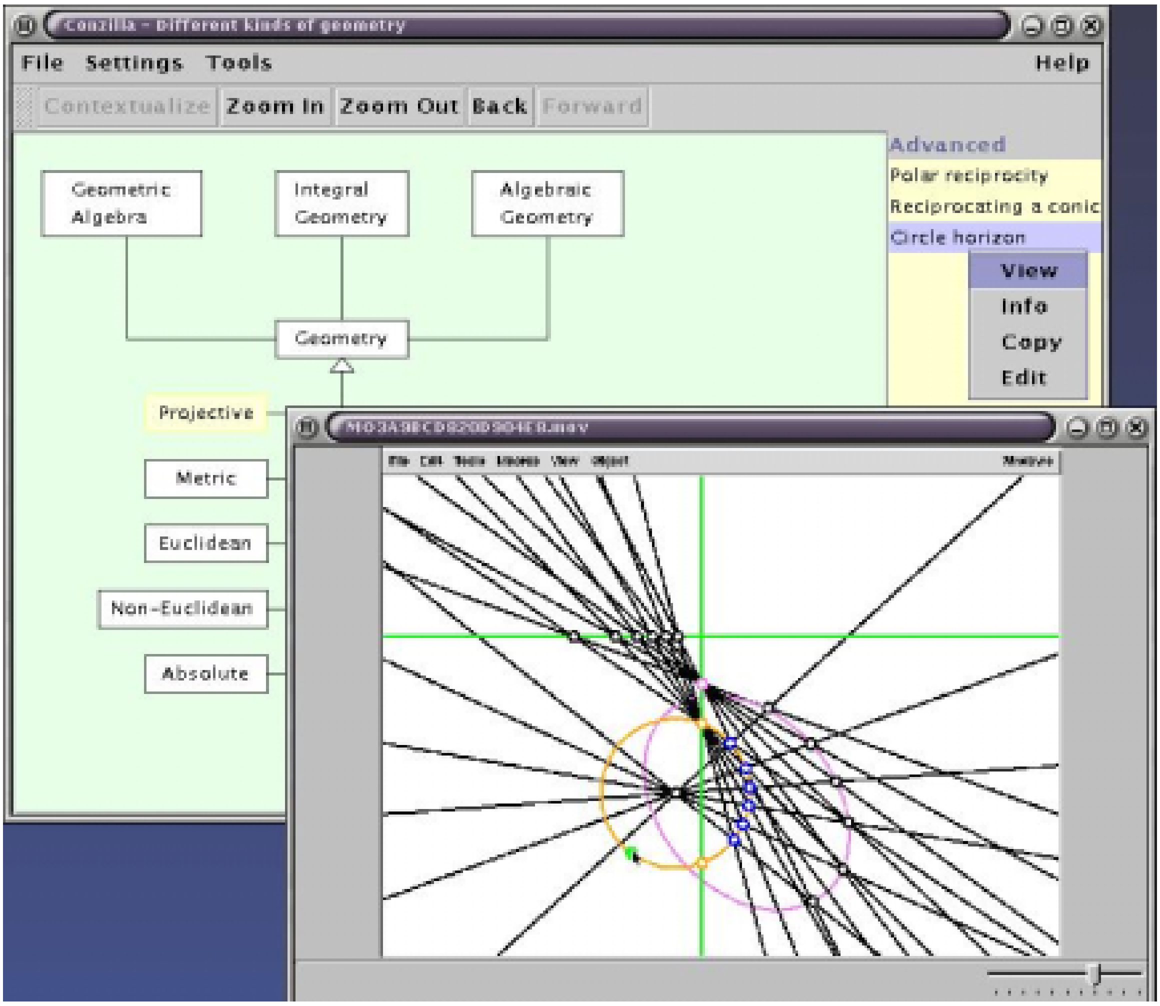

Fig. 9: Pointing to the component "Circle horizon" and choosing "View", displays the content of this component, which in this case is a Quicktime movie. This movie is available at http://www.nada.kth.se/~amb/SnapzPro/Circle_horizon.mov.

6. Displaying the decription of the content components

In Fig. 10 we have displayed the list of content associated with the concept of "Mathematics Learning Environments", which is part of the "Mathematics Environment" context (= map). Note that since there are no filters associated with this collection of components, there are no sub-menus appearing under the corresponding "View" menu. Hence we are looking at the entire list of components associated with the concept "Mathematics Learning Environments".

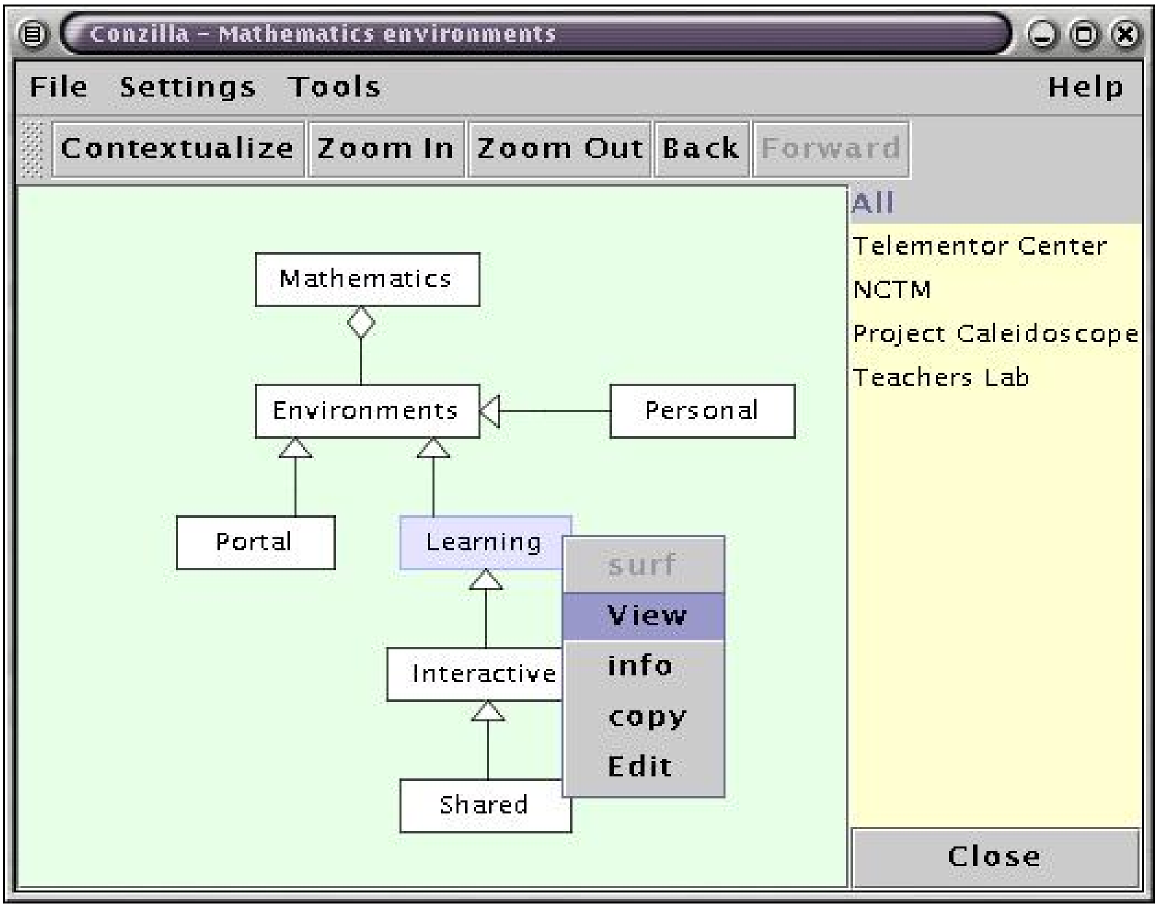

Fig 10: Viewing a list of all the content of "Mathematics Learning Environments".

By pointing to an entry in this list and hitting space-bar (or the → key), the list of metadata descriptions for this entry can be manipulated just as before (Fig. 5).

In Fig. 11 we have displayed part of the descriptions for the components "NCTM" and "Teachers Lab". In this figure we are pointing to the "Teachers Lab" entry in the content list. Therefore this entry becomes high-lighted together with its displayed metadata description. Pointing to the entry "NCTM" would high-light this name together with the description "National Council of Teachers of Mathematics".

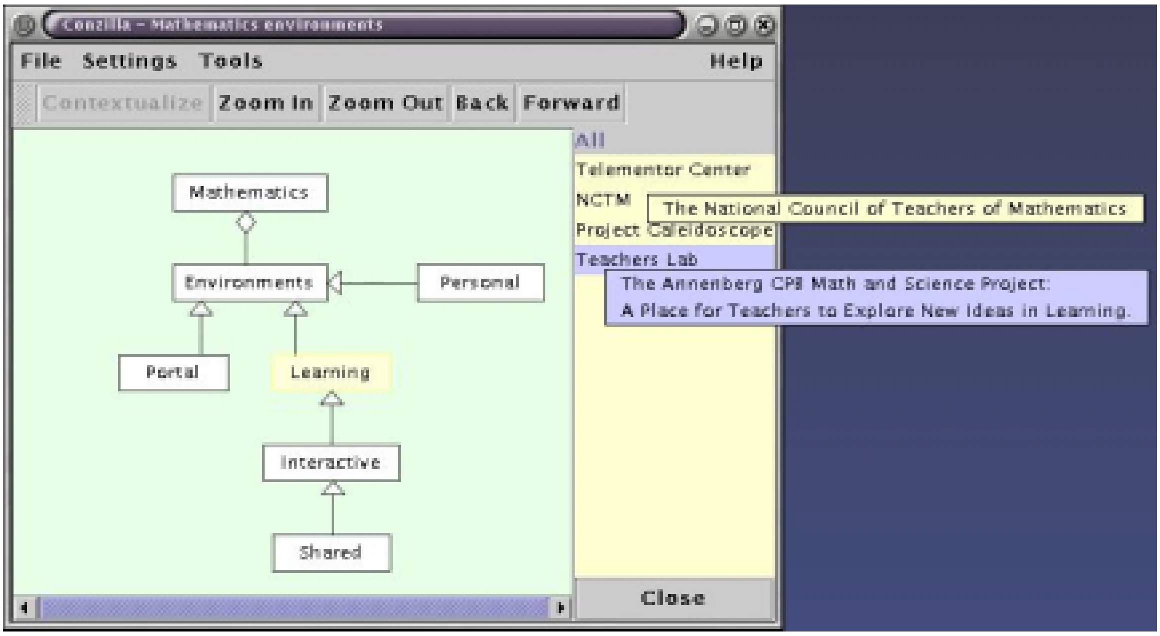

Fig 11: Looking at the descriptions associated with some of the components.

References

[1] Naeve, A., *The Garden of Knowledge as a Knowledge Manifold - A Conceptual Framework for Computer Supported Subjective Education*, CID-17, TRITA-NA-D9708, Department of Numerical Analysis and Computing Science, KTH, Stockholm, 1997. (http://cid.nada.kth.se/sv/pdf/cid_17.pdf)

[2] Naeve, A., *Conceptual Navigation and Multiple Scale Narration in a Knowledge Manifold,* CID-52, TRITA-NA-D9910, Department of Numerical Analysis and Computing Science, KTH, Stockholm, 1999. (http://cid.nada.kth.se/sv/pdf/cid_52.pdf).