Personalized and Shared Mathematics Courselets (MathViz) Progress Report PADLR, May 2002

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Abstract

Students at KTH in Stockholm have trouble with their mathematics courses and difficulties learning certain math concepts. The math visualization courselets and interactive visualization sessions used in this project have been successful and reached a high level of acceptance of methodology by the 1st year mathematics students at KTH. The sessions have beed documented on video and further analysis is continued to develop so called "practitioner tracks" – support for students to understand and use the visualization courselets on their own in between the sessions. During the second year, the experiment will be scaled up to affect all first year mathematics courses.

Introduction

Freshmen students at the IT-university in Kista, Stockholm, and other study programmes at the Royal Institute of Technology in Stockholm seem to have difficulties with both getting used to and with understanding certain mathematics concepts introduced in the math courses of their engineering study programme. They also have trouble seeing the relevance of the math concepts to other subjects in their study programme.
Lecturers in several math and computer science courses have problem carrying out their courses because they are delayed when having to go through more elementary math concepts which were presupposed by the courses. Fewer students pass math courses than any other courses in the IT university study programmes.
The number of students choosing science orientations in general is also decreasing. And the overall number of admitted students at KTH is increasing as well.
The programs need to add extra exercise activities as a support for students. The formal competence of math teachers is excellent, but the study programs cannot afford to pay for student contact hours with math teachers.

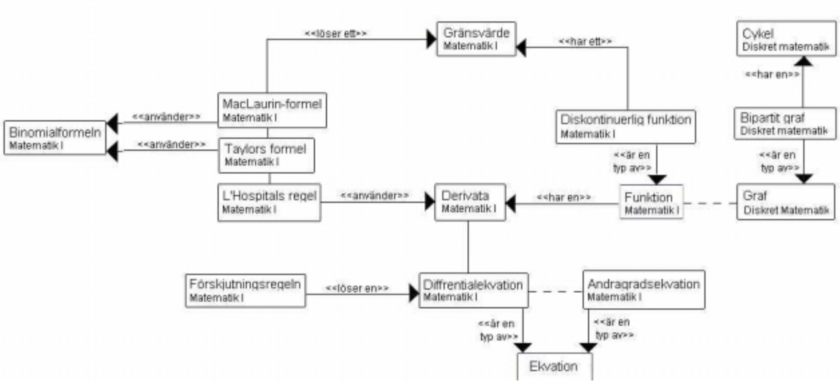
Previous Research

Previously, the mathematical understanding and learning of first year students at the study programmes of the IT University in Kista has been studied in the WGLN project (APE-track A): *Content and context of Mathematics in Engineering Education*. The goal of the project was to try out methods encouraging students to use conceptual modeling to document and reflect on their learning process.
Students seemed to, e.g., have trouble understanding concepts such as Taylor-development, multivariate functions and multivariate equations and also to see when there are, and are not, solutions to math problems. As a first step toward alleviating students' learning problems a study investigating the learning and understanding of math concepts was carried out among the students.
There is extensive research in related areas on design tasks and learning. E.g., "constructionism" – a theory of learning and a strategy for education - suggests a strong

connection between design and learning (Kafai & Resnick, 1996). Related to designing conceptual models are the learning-by-design (LBD) approaches using design challenges as a pedagogical method: ”Construction and trial of real devices would give students the opportunity … to test their conceptions and discover the bugs and holes in their knowledge.” (Kolodner, Crismond, Gray, Holbrook, & Puntambekar, 1998). Others have studies the advantages of using specific diagrams to support conceptual learning (Oshima, Yuasa, & Oshimo, 1998).
We hoped that introducing conceptual modeling techniques and modeling tasks would be beneficial in three different ways, namely:

1. that conceptual modeling would support the learning of mathematical concepts. Engaging students in modeling tasks hopefully supports learners’ conceptual development by making important concept more explicit and by turning learners’ attention to related concepts that they may have neglected otherwise.
2. that conceptual modeling would support and encourage reflection on their learning in general, i.e., that conceptual modeling may be an efficient technique to encourage metacognition. Hopefully students would reflect more on the concepts and the terminology, how they are related, and maybe also about which concepts they had not yet mastered as well as why some of theses concepts were causing learning problems. Understanding why they have problems is the first step towards overcoming learning problems.
3. that conceptual modeling would support “transfer” of math concepts to other (computer science) subjects. Hopefully students would be supported and encouraged to reflect on how the math concepts could be relevant to other subjects.

The study was the first step towards developing and offering students support in their learning. Therefore, the goal was to investigate the learning and understanding of math concepts. 150 engineering students participated in the study involving modeling tasks stretching over the entire first year of the study programme. In the fall, the students initially performed a diagnosis task investigating which concepts they viewed as the most central concepts in mathematics. During the first semester they were asked to construct graphical conceptual models describing and relating all mathematical concepts they were confronted in the study programme. Their views on the math concepts could be expressed in different ways. A Unified Modeling Language (UML) notation was preferred, but if students experienced it as too restrictive other notations were allowed, e.g., “mind maps”. These models were handed in around Christmas, in December. During the spring semester students continued modeling how they viewed the math concepts as well as all new math concepts introduced in the following courses. New models were once again collected in the end of the spring semester. A typical model could look like the one below.



The students’ answers to the diagnosis tasks and the two modeling tasks differed the most. The study showed that students picked up and noted a number of new concepts from the math courses which they included in their models. If the spring models were compared to the fall models a number of differences were also be observed: New concepts were added.

Specifically concepts related to courses the students had attended during the spring semester. The models also become more homogeneous. Perhaps because the students discussed the models with each other, and perhaps because the students became more familiar with UML. The table below shows the most central math concepts according to the students, at three different occasions during their study programme; when initially enrolling in the study programme, after one study semester, and after an entire study year. The lists are based on the number of diagnoses and models in which each concept occurs.

Concept in the diagnosis task	Concepts after one semester	Concepts after one year
Addition	Funktion	Vektor
Subtraktion	Derivata	Matriser
Multiplikation	Gränsvärde	Gränsvärde
Division	Polynom	Determinant
Bråk	Komplext tal	Derivata
Procent	Differentialekvation	Partiell/partialderivata
Variabler	Felrättande koder	Funktion
Potenser	Homogena (ekvationer)	Jacobimatris
Logaritmer	Inhomogena ((diff-)ekv)	Skalärprodukt
Funktioner	Bipartit (graf)	Polynom

In the last model students typically added new concepts from newly attended math courses (e.g., *kedjeregeln, flervariabel, integral, extremproblem*…) to the earlier model. Often students also developed and elaborated on a concept that had previously been included in a model.
Rather often, however, students constructed entirely new models which did not include any parts of their previous models even though much work had been put into these. In many cases students seemed to presuppose the parts not mentioned. Perhaps the parts were left out to save the effort of relating these to new concepts in their new models. Another plausible interpretation is that the connection between courses was not clear enough and concern uses of the certain concepts (e.g., function) which do not overlap thereby making it difficult for students to relate the different uses of the concept in their conceptual models. These connections between courses should perhaps be made clearer and more explicit for the students.

The Personalized and Shared Mathematics Courselets Project

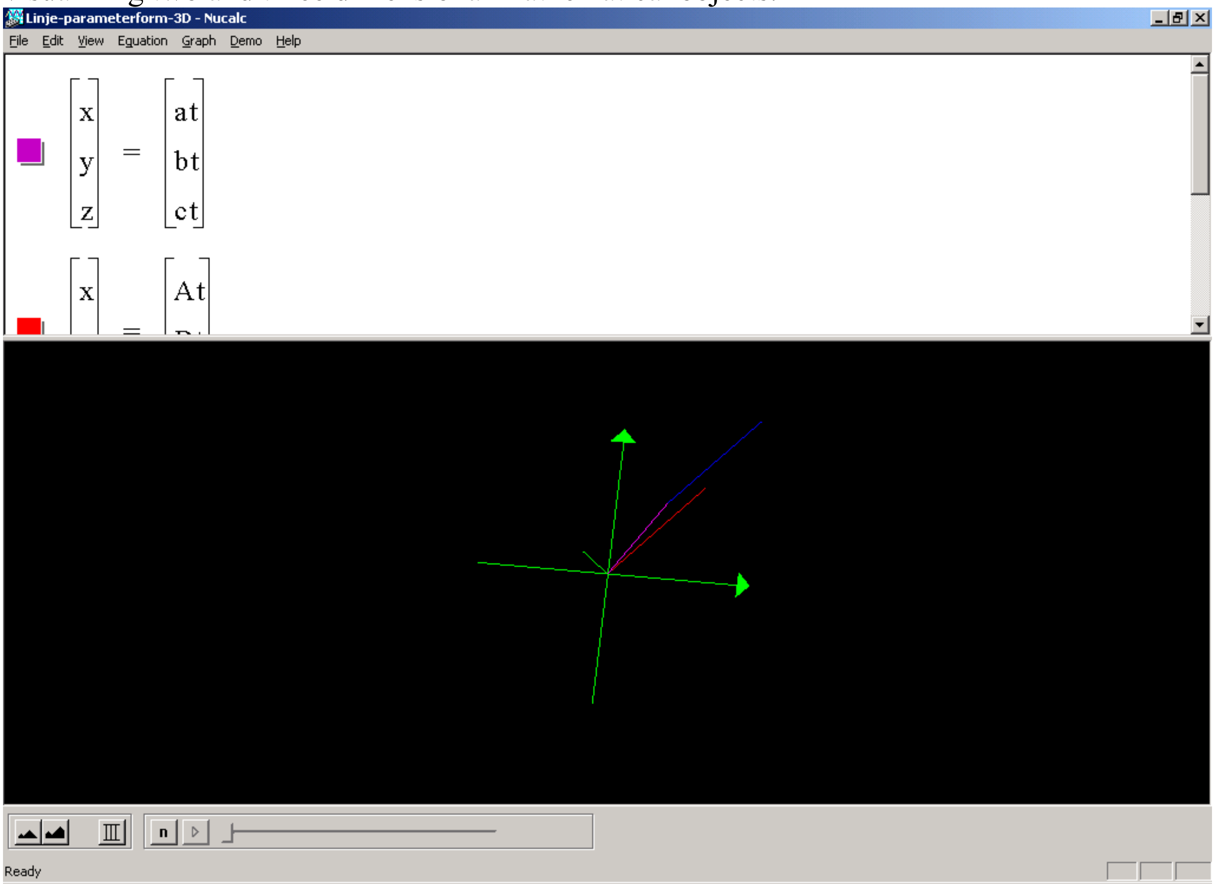
The APE project was followed by the Personalized and Shared Mathematics Courselets project with participants at CID and DSV at KTH. The overall goal of this project has been to develop, launch and evaluate the use of computer-based support for math education on a university level. The point of this has been a pedagogical one, to make abstract concepts that cause problems for students more concrete and understandable through the use of the computer-based tools.

One goal has been to support students and help more of them to pass a mathematics course with a focus on introductory linear algebra and differential multvariate calculus at the study programme (”Mathematics II”).
It has been important for us to have the students feel that the primary goal of the project has been to support their learning and not just engage them as guinea pigs in an experimental research project to evaluate new tools. We have therefore made an effort to present the project as primarily a support for the students.

Initially the plan was to engage students in project work involving the use of the new tools. However, due to a tight schedule, those plans had to be abandoned. Instead students were given the opportunity to participate in visualization sessions where central math problems were discussed an visualized using the Graphing Calculator software. These were carried out weekly during the course. The sessions are planned to continue after the course as a service for those students who do not pass and who need support in mastering the mathematics.

The Graphing Calculator Software

Graphing Calculator (GC) is a graph drawing tool developed by Ron Avitzur, Andy Gooding, Carolyn Wales, and John Zadrozny. The tool can be used for the visualization of and interaction with mathematical expressions. For more info see: www.pacifict.com.
GC is a tool for quickly visualizing math. Users just type an equation and it is drawn for without complicated dialogs or commands. GC features symbolic and numeric methods for visualizing two and three dimensional mathematical objects.



A screen shot of theNuCalc 2.0 software.

It should be pointed out that the GC software has not been developed specifically for the kind of situations we have used it in. The software is not tailored for seminars with fourty students discussing a specific math problem. It is rather developed for individual use and other pedagogical approaches than collaborative problem solving tasks and this fact may restrict the possibilities for successful visualizations sessions.

A Web-Based Repository of Courselets

During the fall of 2001 an archive consisting of a number of mathematics courselets were developed by Ambjörn Naeve. The courselets were developed in GC and illustrate various concepts concerning linear algebra and geometry. The courselets were made available to the students over the web. Students could download the courselets, run them on their own computers and manipulate the examples as they wanted. The GC 3.1 software was purchased and made available to all students.

Interactive Exercise and Visualization Sessions

A mathematics course (Mathematics II) was given during the spring semester 2002 at the IT university. In connection to this course, the visualization sessions were conducted as a support for students beginning February 11, 2002. The sessions, led by Naeve, were carried out each Monday afternoon and was open to students who wished to attend.

First the GC software and then courselets were introduced to the students. After these introductions it was made clear to the students that they were the ones who could influence the direction the sessions would take. Any mathematics problem related to the course or even example in the course literature causing the students problems would be discussed and visualized in the sessions. The point of the visualization sessions was not to go outside the boundaries of the course, the point was rather to help students pass the course. Once the students realized that this was the case, they seemed enthusiastic and participated more and more actively in the sessions. During the course a number of math courselets were discussed. When examination time got closer, examples taken from previous exams were discussed.

Ambjörn Naeve leading a visualization session

Typically, the sessions would last about two hours. But sometimes both the students and the leader of the sessions would be so deeply involved in the math problems being discussed so that the sessions were twice as long. A specific math problem could be discussed and scrutinized in detail for 45 minutes. Between 11 and 41 students participated in the sessions.

Students discussing a math problem at a visualization session together with Ambjörn Naeve and Mikael Nilsson.

Interviews confirmed that the students themselves experienced the mathematics courses as difficult. The students appeared to be satisfied with and appreciative of the sessions and viewed these as a chance for learning the mathematics which was hurriedly presented in the normal lectures.

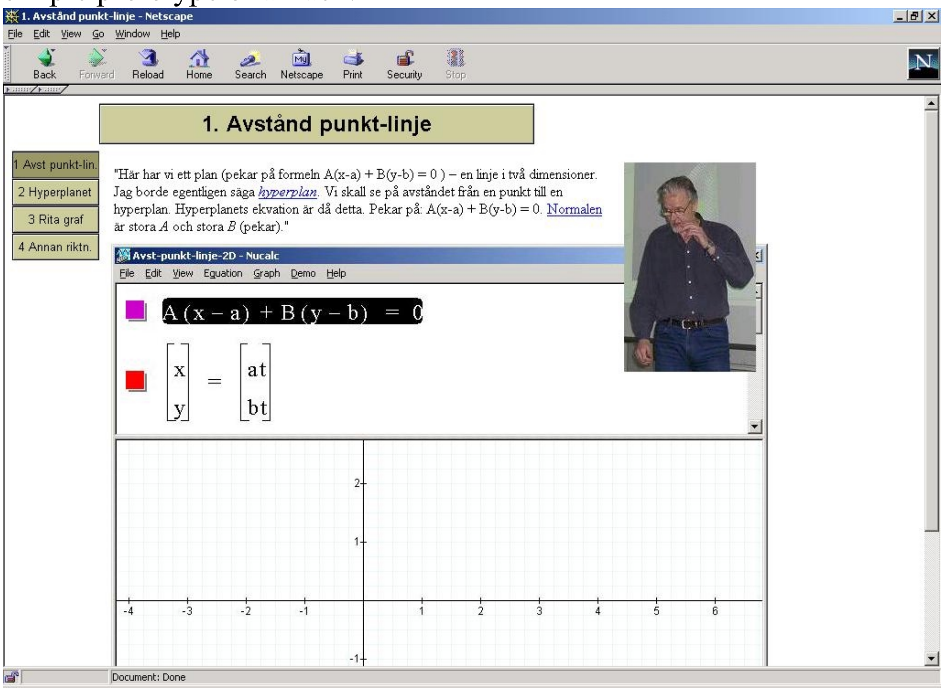
Student Responses to the Visualization Sessions

As mentioned, the students who participated in the visualization sessions seemed to appreciate the support given in the session form. Discussions and interviews with students have given input as to their view of this kind of support.
A typical student remark to the sessions was ”*it is good that you approach the theory in this way*”. Students seemed to appreciate the sessions as a complement to lectures which give an overview of the whole domain but because of the strict time schedule never can dwell as long as needed on all the difficult new concepts. Some of the students also preferred studying on their own to listening to lectures and thereby being able to decide themselves exactly how much time to allocate to the different parts of the course. But since they got stuck on different parts, the visualization sessions were an opportunity to get help with studying the literature.
Another typical student comment to the way math problems were scrutinized in the visualization sessions was that ”*in the lectures a comparable problem is flashed through in ten minutes – we don't have a chance to learn anything*”. The students making these remarks did not do so because they were critical of the quality of the lectures. The point was rather that they needed more help in getting a better and deeper understanding of the concepts, which were rather sketchily overviewed in the lectures. Not until the concepts were discussed and visualized in the visualization sessions did they feel that they got a more concrete understanding of the concepts.
An observation made which was also confirmed in the discussions with the students, was that the sessions brought up and made explicit the mathematical terminology related to the domain. When showing and running the visualizations from the repository of courselets, Naeve would often point what this and that was called and he would mention the names of the mathematical phenomena illustrated in the courselets. These illustrations not seldom turned into discussions about what a term stood for. Sometimes students would point out that a concept had been used in a different manner in the lectures or in the literature: "*the concept 'hyperbola' seems to have different uses within mathematics*". These comments and discussions gave Naeve the possibility to clarify the mathematical terminology and explicate the different related uses of a certain term.
In short, the students were very satisfied with the support that the visualization sessions provided.

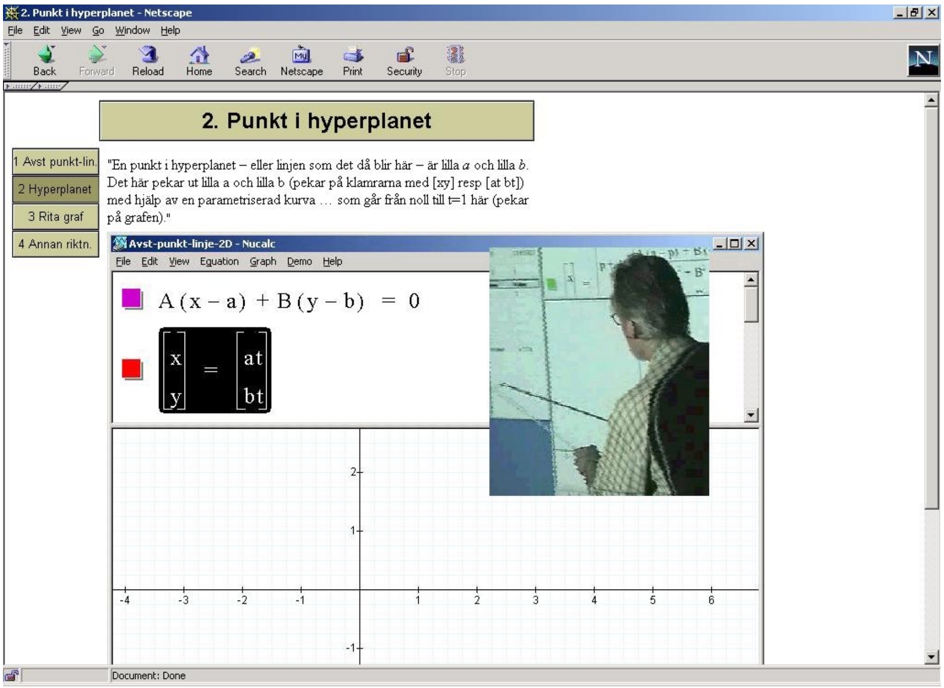
Video Recording of the Sessions

The sessions were recorded on video. This was done for documentation but also for other reasons. The courselets and the GC software obviously have pedagogical value. A problem however remains; if a learner wants to use and learn from the courselets without Naeve or someone else familiar with the mathematical courselets – how can the learner possibly know how to go about to use the courselets in a fruitful way? What does the courselet illustrate? How is it manipulated and interacted with to show an interesting mathematical phenomena? What are the relevant points and results of the visualization? What are the central concepts and terminology related to a particular visualization? Such questions are difficult for a student to answer without someone present to explain the use of the courselets. In the visualization sessions Naeve would typically give elaborate explanations when running through the courselets.

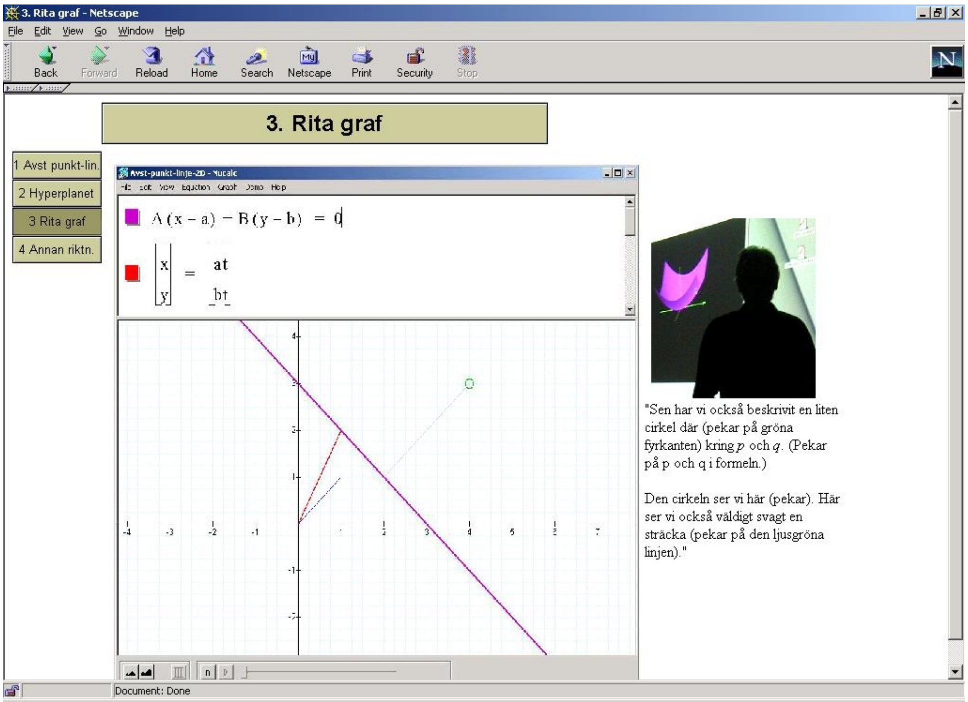
To alleviate some of the mentioned problems and to collect some of the courselet explanations, the sessions were video recorded. The video recordings are used to create so called *practitioner tracks* which show an experienced practitioner (in this case Naeve) illustrating the typical use of a tool (in this case the GC courselets). The practitioner tracks illustrate step by step how the courselets can be used and also represent Naeve's comments to these. The tracks will be available on the web for students to access as a support for learning and using the math courselets and thereby helping them in using the courselets between visualization sessions. The tracks are thus a complement to the visualization tracks and attempt to show what exactly is interesting in the courselets, how they are used, and what concepts are interesting in relation to them.
To add pedagogical value to the tracks and encourage the metacognitive processes of the students, the tracks will have a so called *problem level*. This basically means that the tracks will be designed so that they start out by presenting a problem or question and multiple ways of solving the problem or answering the question (some of which may be incorrrect) *before* presenting a solution and thereby encouraging learners to come up with a solution and not just passively looking at the visualizations. See below for an example of a few steps in a very simple prototype of a track.



1. An example of a prototype for a practioner track illustrating the use of a math courselet. Naeve introduces the problem and explains the conditions and what is seen in the courselet.



2. Naeve goes through and discusses a particular concept which is relevant for the example and also explains the formula which will be the basis for the graph.



3. The graph is finally shown and Naeve discusses details in it and how they relate to the abstract formula.

Continuation of the First Year of the MathViz Project

During the first half year, a number of GC visualization modules have been created and developed. These have been set up as an archive available for the students. In the second half year, the GC-based methodology was introduced in a mathematics course at KTH. Students were able to download and use the pre-authored courselets that were also used in the interactive visualization sessions. The sessions were video-recorded.

In sum, the visualization technique that was used has been very successful and has resulted in a high level of acceptance among the students. The visualization sessions are appreciated among the students who have experienced a need for support in their learning of mathematics.

A continutation of the project is planned in several ways:

- We will continue the further analysis of the visualization sessions which were documented on video. This analysis will result in a report but also in further development of the "practitioner tracks" described above. The tracks are a support for students so that they can learn to use the visualization courselets and also a support for their learning in between visualization sessions.
- The visualization sessions can also continue. New exams in the math courses are organized in the early fall and more visualization support is planned for students. As mentioned above, the intention of the project was initially to engage students in creating their own visualizations. In future courses this can be carried out. Students will work in groups and create visualizations, which can then be made available for the rest of the student group.
- Other tools to support students' learning of mathematics – especially the Conzilla concept browser - will be used.

Continuation of the MathViz Project during the Second Year

Due to the acceptance of the methodology by 1st year mathematics students at KTH which we managed to establish during year 1, we will be able to scale up experiments during the second year, so that they will affect all mathematics courses during year one. We will still introduce the methodology as a complementary support for students who experience problems with certain mathematical concepts. For the last part of year two we will conduct a more substantial empirical study which not only will study the students' acceptance of the methodology, but also the students ability to reuse, adapt and personalize modules within and among courses. The larger scale application will also make possible to detect potential positive changes for the students' examination results. Another development during year 2 will be to utilize the Edutella tools more systematically in the applications.
In the fourth half year, evaluations will be performed, guidelines for the methodology will be developed and documented.

References

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