ICT ENHANCED MATHEMATICS EDUCATION

IN THE FRAMEWORK OF A KNOWLEDGE MANIFOLD

Ambjörn Naeve and Mikael Nilsson

KMR (Knowledge Management Research) group
CID (Centre for user oriented Information technology Design)
NADA (Department of Numerical Analysis and Computer Science)
KTH (Royal Institute of Technology)
100 44 Stockholm, Sweden

{amb | mini}@nada.kth.se

ABSTRACT

This paper presents the *Knowledge Manifold* educational architecture and a number of tools (developed by the KMR group), which enable the construction of interactive mathematics learning environments that support a learner-centric, interest-oriented form of "knowledge pull". They include the *Virtual Mathematics Laboratorium*, constructed with the *Conzilla concept browser*, the *Mathemagic™ portfolio* built in the *SCAM portfolio*, and the *CyberMath* shared 3D virtual reality environment. The pedagogical aim of these tools is to increase the cognitive contact with mathematics, e.g. by modeling its contexts, visualizing its concepts and interacting with the geometrical forms behind its formulas.

1. Introduction and background

It is well known that present mathematics education suffers from serious problems. Prominent among them is the increasing difficulty to motivate students and maintain the interest in the subject that is almost always present at a very young age but which seems to diminish – and often totally disappear - as the years go by. Most educational system of today are based on closed, layered architectures of different levels (elementary, intermediate, secondary, high-school, university) with almost no contact between them – especially between the non-adjacent ones. Moreover, in mathematics, the teachers at the early levels often suffer from a lack of understanding of the real nature of the subject – and e.g. often confuse mathematics with arithmetic – while the teachers at the later (university) levels often suffer from a lack of pedagogical interest, which results in various efforts to minimize their teaching 'duties'.

Other shortcomings of the traditional mathematics education architecture include its inability to: stimulate interest, promote understanding, support personalization, facilitate transition between different layers, integrate abstractions with applications, and integrate mathematics with human culture. During the last 20 years, Ambjörn Naeve has developed a pedagogical

approach to mathematics education that makes use of ICT in order to address these problems [4]. In various experiments[1], we have used this approach to demonstrate that it is possible to increase the "cognitive contact" with mathematics in different ways, such as e.g. by clearly expressing the mathematical contexts as well as by visualizing the mathematical concepts and interacting with the forms behind the mathematical formulas. We have also experimented with enhancing the mathematical narrative[2] by showing before proving, proving only when the need for a proof is obvious to the students, and focusing on the historical development of mathematics from a philosophical and history-of-ideas perspective[3].

2. The Knowledge Manifold

A *knowledge manifold* [3], [5], is an information architecture, which highlights the complementarity of *context* and *content* and supports a variety of different strategies for context-dependent presentation and suppression of information. It consists of a number of linked information landscapes (contexts), where one can navigate, search for, annotate and present all kinds of electronically stored information. A Knowledge Manifold consists of a number of linked *knowledge patches* - each maintained by a custodian called a *knowledge gardener*. A knowledge patch in turn consists of a set of resource components[4] that are tied together with *context-maps* that represent the corresponding conceptual model of the subject domain. Such context-maps are preferably constructed using the *Unified Modeling Language* [12], which is an industry standard for this purpose.

When used for learning purposes, the KM architecture supports the following seven different knowledge roles[5]:

- the *knowledge cartographer*, who constructs and maintains context-maps.
- the *knowledge librarian*, who fills context maps with content.
- the *knowledge composer*, who constructs customized learning modules.
- the *knowledge coach*, who cultivates questions.
- the *knowledge preacher*, who provides live answers.
- the *knowledge plumber*, who directs questions to appropriate preachers.
- the *knowledge mentor*, who is a role model and supports self-reflection.

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[1] http://kmr.nada.kth.se/math/index.html

[2] i.e. to tell more interest-provoking mathematical stories.

[3] For more information on these experiments, see http://kmr.nada.kth.se/proj/ape.html#apea and http://kmr.nada.kth.se/proj/ape.html#apec.

[4] In KM terminology, we use the term *knowledge component*, when we assume the perspective of the teacher(s), the term *information component* (or *learning component*) when we assume the perspective of the learner(s), and the term *resource component* when want to remain neutral in this respect. It is the *transformation* of learner information into learner knowledge, and eventually the *transmutation* of learner knowledge into learner understanding, that is the overall aim of the learning process.

[5] See [5] for a further discussion of these knowledge roles.

It is fundamentally important that all these roles should be available to both teachers and learners[6]. "You learn as long as you are teaching" is the pedagogical principle at work here.

The KMR group at CID is making use of the knowledge manifold architecture in the construction of interactive learning environments that enable a learner-centric, interest-oriented form of "knowledge pull", and which support inquiry-based and personalizable forms of networked learning. An important design goal for these learning environments is to support the transformation of the teaching role - away from the traditional "knowledge filter" towards more of a "knowledge coach", i.e. away from "teaching you what I know" and towards "helping you to find out more about the things that you are interested in". We believe that this is a way to create sustainable solutions to the current educational crisis.

3. Conzilla – the concept browser

A *concept browser* [6] is tool that supports the exploration and presentation of the information stored in a Knowledge Manifold. *Conzilla* [9] is a first prototype of a concept browser, which has been developed by the KMR-group since 1998. Conzilla makes it possible to structure and present electronically stored information in ways that improve the overview and enhance the structural connections between the different parts. Through user-feedback the interface has matured and improved, and Conzilla is now starting to attract serious attention both on the national and the international level.

Our mathematics education work includes the following projects:
- Using Conzilla to construct a *Virtual Mathematics Laboratorium* in the form of a knowledge manifold [4].
- Exploring new ways to study geometrical constructions by doing interactive geometry with *PDB* (*Projective Drawing Board*) [15].
- Studying new ways to interact with mathematical formulas, using programs like the *Graphing Calculator* [1], [16].
- Building mathematical components archives in the *SCAM portfolio* [7], [11] and developing methods to interact with these components over the *Edutella* network [14][7].
- Exploring different ways to interact with mathematics in shared 3D interactive learning environments by using *CyberMath* [13].

[6] Note that when maintaining his or her knowledge patch, the gardener performs the role of cartographer when working with the contexts and the role of librarian when working with the content.
[7] See [10] for a further discussion on the importance of Edutella.

4. The Virtual Mathematics Laboratorium

We will now take a brief look at the *Virtual Mathematics Laboratorium* (VML), which is a mathematical knowledge manifold[8] constructed in Conzilla. The mathematical concepts are described with metadata and filled with content-components. Currently, the VML contains more than 600 different interactive mathematics components (learning objects)[9].

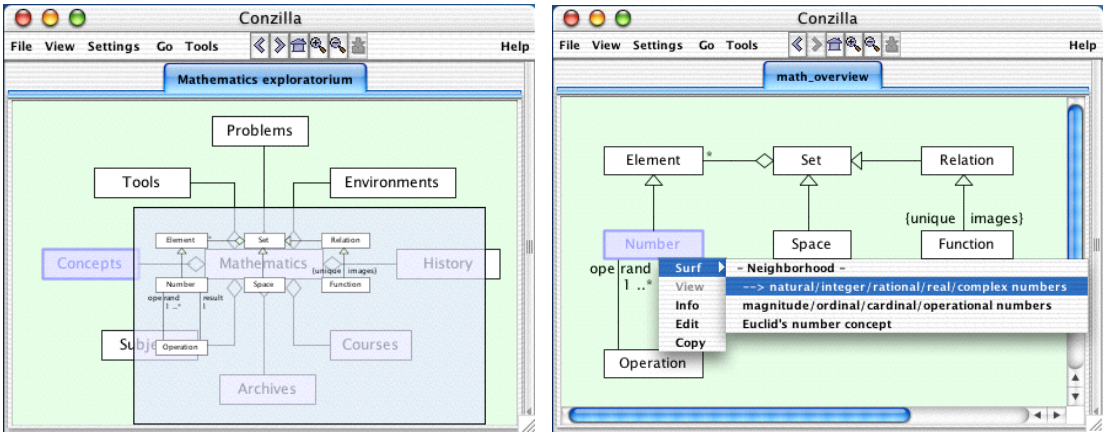

Figure 1. Left: The entry to the VML, showing the detailed map of `Mathematics Concepts`. Right: Surfing the contextual neighborhood of the `Number` concept

A UML diagram [12] of the entry to the VML is shown to the left of Figure 1, where the `Mathematics Concepts` box has been left-clicked in order to bring up an icon of its detailed map. The idea is to give the visitor a glimpse of this more detailed context-map without having to move into it. Double clicking the `Concepts` box takes the visitor into this context-map (shown to the right), which contains an overview of some of the most important concepts of mathematics and how they are related[10]. Right-clicking the `Number` concept brings up a menu which lets the visitor switch contexts in another way. Clicking on `Surf` provides access to the *contextual neighborhood* of the `Number` concept, which is the collection of all other context-maps where this concept appears. Choosing the context-map, which is highlighted in the menu, brings the visitor into the context-map of Figure 2. This figure shows a context-map of the most common kinds of numbers in mathematics and the relationships between these kinds of numbers. More precisely, the map show that a `Natural Number` is a kind of `Integer Number`, which is a kind of `Rational Number`, which is a kind of `Real Number`, which is a kind of `Complex Number`.

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[8] See [4] for a more detailed discussion of the VML.

[9] About 400 of these learning objects have been created by Ambjörn Naeve. They are also available through his *Mathemagic™ SCAM portfolio* [7], which is freely licensed for non-commercial use under a *Creative Commons* license (http://creativecommons.org).

[10] According to the gardener of this knowledge patch, which is Ambjörn Naeve.

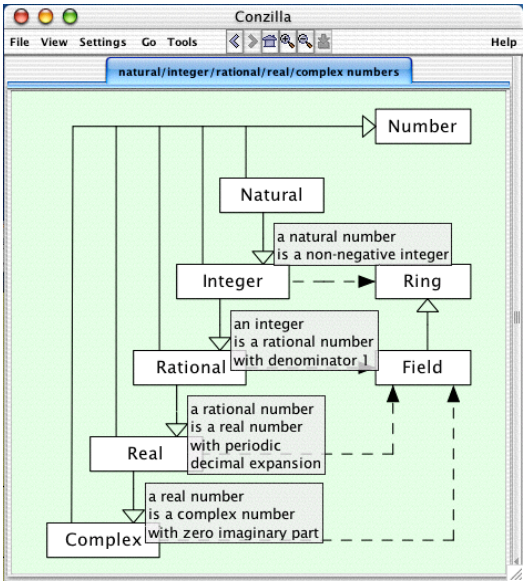

Figure 2. Different kinds of numbers with exposed metadata describing their relationships

In Figure 2, some of the metadata on these relationships have been exposed, which gives more detailed information about how these different kinds of numbers are related. It is important to observe that this metadata belongs to the *relations* (= arrows) between the boxes that represent the different number-types. The metadata has been exposed by pointing to the respective relations (one-by-one), right clicking them and selecting the `Info` command from the pop-up menu that is visible in Figure 3. In this mode, Conzilla works as a non-linear presentation tool – as opposed to e.g. PowerPoint™, where the presentations are linear – i.e. totally ordered with no possible changes at 'run-time'. The context-map also shows that `Integer` is an example of `Ring` and that `Rational`, `Real` and `Complex` are examples of `Field`, which in turn is a special kind of `Ring`. This example shows the open, layered architecture of the VML, where a curious 10-year old could suddenly find herself transported into a traditional university context.

Right-clicking the concept `Complex (Number)` and selecting the `View` command opens the *content window* (to the right in Figure 3), which displays a list of content-components that have been associated with the `Complex` concept. In this case there is only one component – named `Mathemagic archive`, and exposing its metadata shows that it is "Ambjörn's component archive using the SCAM portfolio". Clicking on `Mathemagic archive` brings up the `Complex` part of this archive in an ordinary web browser – in this case Mozilla - as shown in Figure 3. Now we are ready to dig into the content of this part of the archive, but – in contrast to the experience of ordinary surfing – we can do so without losing the overview of the original context.

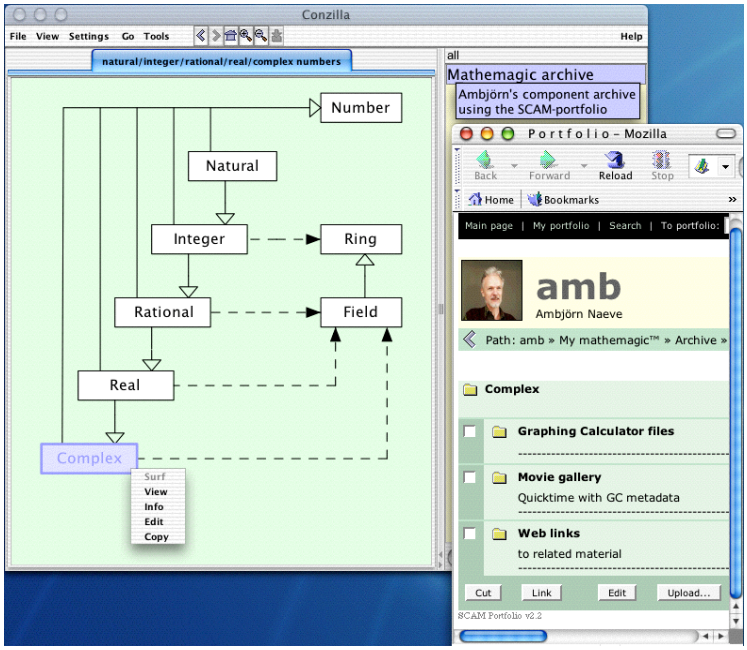

Figure 3. Clicking on "Mathemagic archive" opens the Complex (number) part of Ambjörn's mathemagic SCAM portfolio in an ordinary web browser

5. CyberMath

Figure 4. Avatars visiting the solar energy room in Cybermath

A different kind of interactive learning environment has been created in the *CyberMath* project [13]. CyberMath is an experimental 3D program for interacting with mathematical objects in a virtual reality environment. CyberMath is also a collaborative space where participants from different physical locations, represented by *avatars*, can enter the space and take part in lectures, demonstrations etc., while sharing both sound and sight of each other's actions. CyberMath is presently being integrated with Conzilla, so that

objects in the virtual space can be linked to external descriptions available on the *Semantic Web*, the emerging next generation of the Internet. This will open up a world of related information to users of Cybermath.

6. Pedagogical experiments

The tools described above have been tested in a number of pedagogical experiments with ICT enhanced mathematics education – with enthusiastic reactions from most of our 'guinea-pig' students. We have tested CyberMath with first year university students, both as a stand-alone application, and complemented with a virtual presence production system in order to increase the level of face-to-face contact [2]. We have used the Mathemagic SCAM portfolio in conjunction with the Graphing Calculator, in order to create personalized math courselets for first year university students [1], and we have used the Graphing Calculator as a way to overcome "mathematical neurosis" with students at the secondary school level [16].

7. Conclusion

As discussed more fully more in [8], the KMR group has created an infrastructure, a learning architecture and a set of ICT tools that open new possibilities for learner-centric, interest-oriented knowledge pulling learning environments on a global scale. The Swedish Educational Broadcasting Company has adopted the SCAM framework as a basis for its digital media library, and will work with the SCAM portfolio in order to create customized learning modules in a number of subjects, including mathematics. We see this development as an important step towards creating a global math "rehabilitation clinique", where people can get help to restore their confidence in their ability to apply their logical thinking abilities in order to appreciate the inherent beauty of mathematics.

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